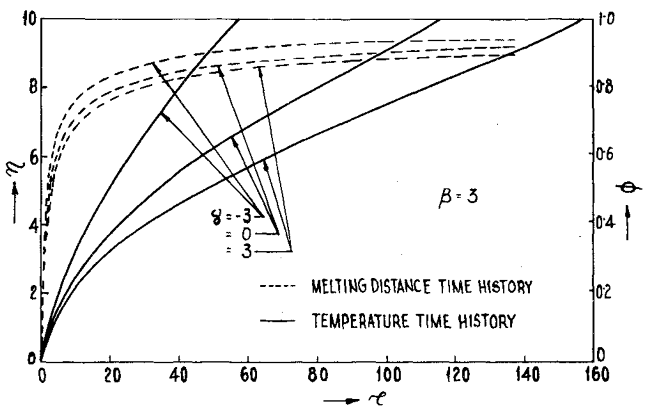

Fig. 2 Effect of variation in γ on melting distance time history and surface temperature time history for a particular value of β.

Using the nondimensional form of Eq. (14), Eq. (17) in terms of melting distance is:

$$\tau=\eta^2/2+\eta \tag{18}$$

This is the same expression obtained by the heat balance integral method[8] using parabolic temperature profile in the melt and Biot's variational method[4] using linear temperature profile [Eq. (5)].

To find the numerical solution, Eq. (15) is numerically integrated between the limit of Φ varying from 0 to 1 using Simpson's formula and the limit is divided into 100 parts with the value of each part being equal to 0.01. This yielded reliable results.

Figure 1 presents surface temperature $\phi(\tau)$ and melting distance $\eta(\tau)$ as a function of time with γ as a parameter for different values of β which, for quartz and graphite used as ablating materials,[11] become, respectively, 6.05×10^{-4} and 1.244×10^{-4} per degree Kelvin rise in temperature above their respective melting temperature. It shows that both of them increase with an increase in time for any β, whereas they decrease with increase in β at any time. Similar behavior was also observed for other values of γ. Figure 2 illustrates the time-variant surface temperature $\phi(\tau)$ and melting distance $\eta(\tau)$ for variable heat capacity for a particular value of β. Negative and positive γ correspond, respectively, to decrease and increase in heat capacity with rise in temperature, whereas $\gamma=0$ represents the heat capacity invariant with change in temperature. It is seen that as γ changes from -3 to $+3$ more time is required to obtain the same $\eta(\tau)$ and $\phi(\tau)$ at a higher value of γ. This is true for all values of β except $\beta=0$ for which these predictions do not depend on γ, as is evident from Eqs. (17) and (18).

Conclusion

A variational method of solution has been employed to predict the surface temperature time history and melting distance time history during ablation of melting solids due to aerodynamic heating. It takes into account the effect of temperature-dependent heat capacity of solids on ablation.

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Unsteady Three-Dimensional Compressible Stagnation-Point Boundary Layers

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Introduction

THE solution of many practical problems in fluid dynamics requires the understanding of unsteady three-dimensional compressible boundary layers. The unsteady laminar two-dimensional and axisymmetric compressible boundary layers for constant $\rho\mu$ flows ($\rho\alpha T^{-1}$, $\mu\alpha T$, $\sigma=$constant where ρ, σ, μ, and T are the density, Prandtl number, viscosity, and temperature, respectively), when the incident freestream varies arbitrarily with time, have been studied by several authors[1-5] using momentum-integral or series-expansion methods. Recently, Vimala and Nath[6] restudied the two-dimensional stagnation-point flow problem for constant $\rho\mu$ flows using an implicit finite-difference scheme.

In the present Note, the unsteady three-dimensional laminar compressible stagnation-point boundary layers for variable $\rho\mu$ flows ($\rho\alpha T^{-1}$, $\mu\alpha T^{\omega}$, $\sigma=$constant where ω is the index of the power-law variation of viscosity) have been studied when the wall temperature and the incident stream vary arbitrarily with time. The equations governing the flow have been solved numerically using an implicit finite-difference scheme.[6,7] Computations have been carried out for the incident stream which moves with constant acceleration and fluctuates about a steady mean, and for the wall temperature decreasing with time.

Governing Equations

The governing equations in dimensionless form for the unsteady laminar compressible boundary-layer at a three-dimensional stagnation point for variable $\rho\mu$ flows, under the assumptions that 1) the incident stream and wall temperature vary arbitrarily with time, 2) the dissipation terms are negligible at the stagnation point, and 3) the external flow is

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homentropic, are given by[6,8,9]

$$(NF_\eta)_\eta + \phi[(f+cs)F_\eta + g - F^2] + \phi^{-1}\frac{d\phi}{dt^*}(g-F) - F_{t^*} = 0 \quad (1a)$$

$$(NS_\eta)_\eta + \phi[(f+cs)S_\eta + c(g-S^2)] + \phi^{-1}\frac{d\phi}{dt^*}(g-S) - S_{t^*} = 0 \quad (1b)$$

$$\sigma^{-1}(Ng_\eta)_\eta + \phi(f+cs)g_\eta - g_{t^*} = 0 \quad (1c)$$

The boundary conditions at time t^* are:

$$F = S = 0,\ g = g_w\bar{\phi}(t^*) \text{ at } \eta = 0$$
$$F \to 1,\ S \to 1,\ g \to 1 \text{ as } \eta \to \infty \quad (2)$$

where

$$\eta = (\rho_e a/\mu_e)^{1/2}\int_0^z (\rho/\rho_e)dz,\ c = b/a,\ F = f_\eta,\ S = s_\eta \quad (3a)$$

$$u = u_e F,\ v = v_e S,\ u_e = ax\phi(t^*),\ v_e = by\phi(t^*)$$
$$w = -(\rho_e/\rho)(\mu_e a/\rho_e)^{1/2}[(f+cs)\phi(t^*) + \eta_{t^*}] \quad (3b)$$

$$h/h_e = g,\ N = \rho\mu/\rho_e\mu_e = g^{\omega-1} \quad (3c)$$

Here x,y,z are the principal, transverse and normal directions, respectively; u,v,w are the velocity components along the x,y,z directions, respectively; η is the similarity variable; t^* is the dimensionless time; h and c are the specific enthalpy and the ratio of velocity gradients along the y and x directions, respectively; F and S are the dimensionless velocity components along the x and y directions, respectively; g is the dimensionless enthalpy; N is the ratio of the density-viscosity product across the boundary layer; ϕ and $\bar{\phi}$ are arbitrary functions of time t^* representing the nature of unsteadiness of the external stream and wall temperature, respectively; the subscripts e and w denote conditions at the edge of the boundary layer and at the wall, respectively; and the subscripts η and t^* denote partial derivatives.

It is assumed here that the flow is steady at first and then changes to an unsteady flow at $t^* > 0$. Therefore, the initial condition for F, S, and g at $t^* = 0$ are given by the steady-flow equations obtained by putting

$$\phi(t^*) = \bar{\phi}(t^*) = 1,\quad \frac{d\phi}{dt^*} = 0,\quad \frac{\partial}{\partial t^*} = 0 \quad (4)$$

in Eqs. (1) and (2). The steady-flow equations are the same as those of Refs. 8 and 9. The parameter c represents the nature of the three-dimensional stagnation points. For nodal points of attachment $c \geq 0 (0 \leq c \leq 1)$ and for saddle points of attachment $c < 0(-1 \leq c < 0)$. Also, $c = 0$ and 1 for two-dimensional and axisymmetric stagnation-point flows, respectively. For high-temperature flows, $\omega = 0.5$, for low-temperature flows, $\omega = 0.7$, and $\omega = 1$ represents the constant density-viscosity product simplification.[9]

The skin-friction coefficients along the x and y directions and the heat-transfer coefficient in terms of Stanton number St are given by[8,9]

$$C_f = 2\tau_x/\rho_e(u_e^2)_{t^*=0} = 2(Re_x)^{-1/2}\bar{g}_w^{\omega-1}\phi(t^*)(F_\eta)_w,$$
$$\bar{g}_w = g_w\bar{\phi}(t^*) \quad (5a)$$

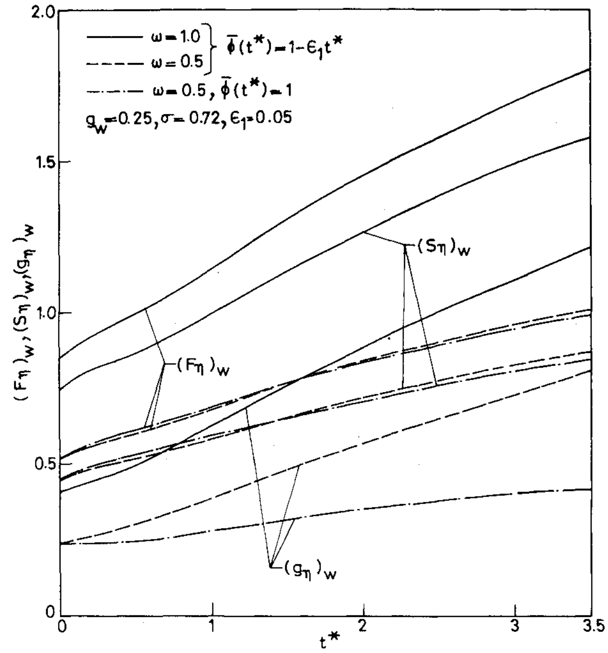

Fig. 1 Skin-friction and heat-transfer parameters for $\phi(t^*) = 1 + t^*$ and $c = 0.5$.

$$\bar{C}_f = 2\tau_y/\rho_e(u_e^2)_{t^*=0} = 2(Re_x)^{-1/2}(v_e/u_e)\bar{g}_w^{\omega-1}\phi(t^*)(S_\eta)_w,$$
$$Re_x = ax^2/N_E \quad (5b)$$

$$St = q_w/[(h_e - h_{w0})\rho_e(u_e)_{t^*=0}]$$
$$= (Re_x)^{-1/2}\sigma^{-1}(1-g_w)^{-1}g_w^{\omega-1}(\bar{g}_\eta)_w \quad (5c)$$

where C_f and $\bar{C}_f$ are the skin-friction coefficients along the x and y directions, respectively; τ_x and τ_y are the wall shear stresses along the x and y directions, respectively; q_w is the wall heat-transfer rate; $(F_\eta)_w$ and $(S_\eta)_w$ are the skin-friction parameters at the wall along the x and y directions, respectively; $(g_\eta)_w$ is the heat-transfer parameter at the wall; h_{w0} is the specific enthalpy at the wall when $t^* = 0$; and Re_x is the local Reynolds number.

Results and Discussions

The set of equations (1) under conditions (2) and with relations (4) have been solved numerically using an implicit finite-difference scheme which is described in great detail in Refs. 6 and 7. Although computations have been carried out for various values of parameters c, ω, and g_w, for the sake of brevity, results for $c = \pm 0.5$, $g_w = 0.25$, and $\omega = 1$ and 0.5 have been presented here.‡ The unsteady freestream velocity and wall temperature distributions§ studied here are

$$\phi(t^*) = 1 + t^*,\ 1 + \epsilon\sin(\omega^* t^*);\quad \bar{\phi}(t^*) = 1 - \epsilon_1 t^*$$

where ϵ and ϵ_1 are small positive constants and ω^* is the frequency parameter. It may be noted that more general variations of $\phi(t^*)$ and $\bar{\phi}(t^*)$ with t^* can easily be included in the analysis.

The skin-friction parameters along principal and transverse directions $(F_\eta)_w$ and $(S_\eta)_w$, and the heat-transfer parameter $(g_\eta)_w$ for $\phi(t^*) = 1 + t^*$ are given in Figs. 1 and 2 for $\phi(t^*) = 1 + \epsilon\sin(\omega^* t^*)$ in Figs. 3 and 4. It is evident from Figs. 1 and 2 that $(F_\eta)_w$, $(S_\eta)_w$ and $(g_\eta)_w$, in general, increase with

‡The results for other values of parameters may be obtained from the authors.

§The detailed results for constant-wall temperature ($\bar{\phi}(t^*) = 1$) will not be presented here.

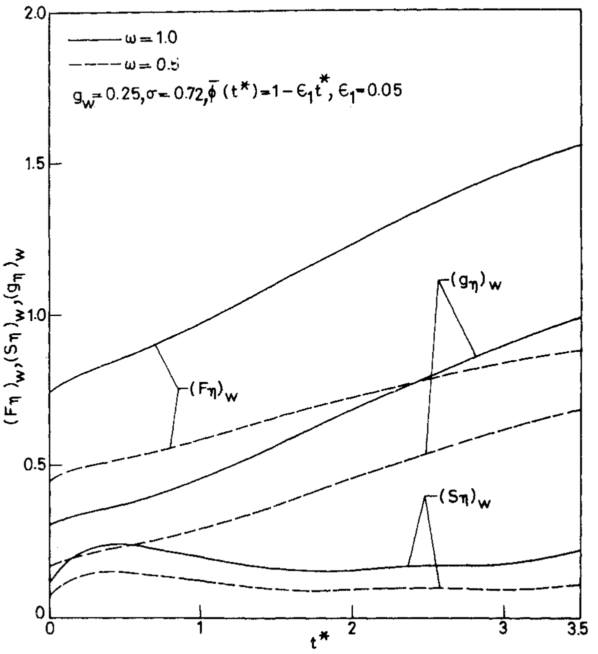

Fig. 2 Skin-friction and heat-transfer parameters for $\phi(t^*) = 1 + t^*$ and $c = -0.5$.

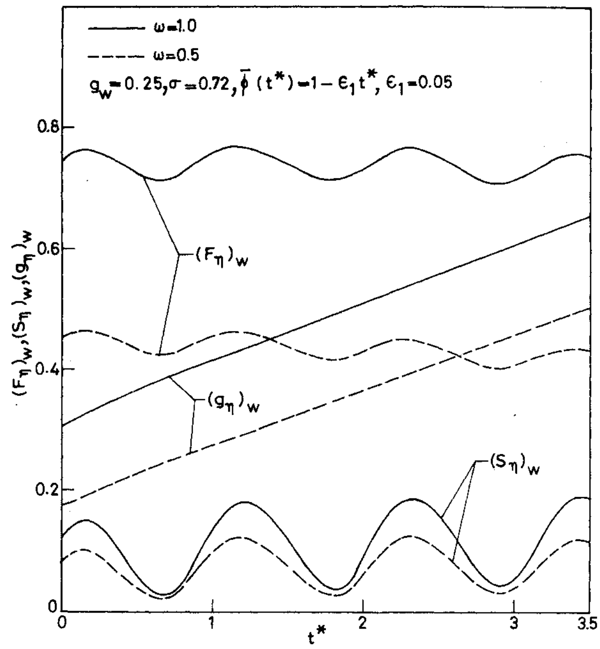

Fig. 4 Skin-friction and heat-transfer parameters for $\phi(t^*) = 1 + \epsilon \sin(\omega^* t^*)$ and $c = -0.5$ ($\epsilon = 0.1$, $\omega^* = 5.6$).

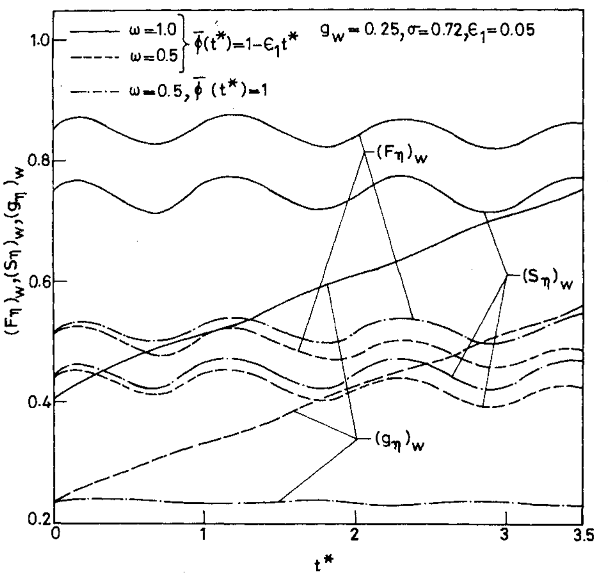

Fig. 3 Skin-friction and heat-transfer parameters for $\phi(t^*) = 1 + \epsilon \sin(\omega^* t^*)$ and $c = 0.5$ ($\epsilon = 0.1$, $\omega^* = 5.6$).

t^* whatever the value of ω may be except $(S_\eta)_w$, which for $c = -0.5$ tends to oscillate first and then slowly increase with t^*. For a given t^*, $(F_\eta)_w$, $(S_\eta)_w$ and $(g_\eta)_w$ decrease as ω decreases. The effect of the variation of the wall temperature with t^* is more pronounced on $(g_\eta)_w$ than on $(F_\eta)_w$ and $(S_\eta)_w$. It is seen that for all values of ω, g_w, and t^*, $(F_\eta)_w$, $(S_\eta)_w$, and $(g_\eta)_w$ decrease as c decreases, until at some negative value of c, $(S_\eta)_w$ is reversed and $(F_\eta)_w$ and $(g_\eta)_w$ begin to increase as c decreases. Similar behavior has also been observed by Libby[8] and Wortman et al.[9] for the steady-state case.

From Figs. 3 and 4, it can be concluded that $(F_\eta)_w$ and $(S_\eta)_w$ respond more to the fluctuations of the freestream velocity than $(g_\eta)_w$, whereas $(g_\eta)_w$ is more sensitive to the variation of the wall temperature with time. Furthermore, the results are strongly dependent on ω.

We have compared our steady-state results ($t^* = 0$) with those of Libby[8] and Wortman et al.[9] and unsteady-state results ($t^* > 0$) for $c = 0$ (two-dimensional flow), $\omega = 1$, $g_w = 0.5$, $\bar{\phi}(t^*) = 1$ (constant-wall temperature) with those of Vimala and Nath[6] and have found them in excellent agreement.

Conclusions

The skin friction and heat-transfer parameters are strongly affected by the variation of the density-viscosity product across the boundary layer and by the nature of the stagnation point. The effect of the variation of the wall temperature with time on the heat-transfer parameter is appreciable; whereas its effect on skin-friction parameters is comparatively small. Furthermore, the skin-friction parameters respond more to the fluctuations of the freestream velocity than the heat-transfer parameter.

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